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Numerical Solution for Non-Stationary Heat Equation in Cooling of Computer Radiator System

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Abstract

In the paper a mathematical model is developed by using numerical solution to study the non stationary heat equation in one dimension. This model leads to incorporate a cooling system for a computer radiator. The result of the model is compared with the stationary heat transfer model. The simulation results show the performance of non stationary model in terms of variation of temperature.

Keywords Cooling System Finite Difference Modelling Simulation Non-Stationary Heat Equation Dimensionless Linear Partial Differential Equation (LPDE)

Introduction

For cooling the semiconductor device of a computer system, there are various known techniques, such as thermal conduction or air-cooling, or the use of a heat pipe, or liquid cooling [1,2]. It is typical to use a heat sink with the central processing unit of a computer to increase the heat-dissipating surface area of the central processing unit for more effective cooling. The model used in this work is based on air cooling by non stationary heat equation in one dimension.

Partial differential equation is a relation involving an unknown function of several independent variables and its partial derivatives with respect to those variables [3]. In recent years seeking exact solution of linear partial differential equation is of great significance as it appears that the LPDEs are mathematical models of complex physics phenomena arising in physics, mechanics and engineering. The simulation of the developed model is tested by using finite difference methods [1]. The advantages of this model are low computation, low complexity of the implemented algorithm and optimization of model efficiency. To analyze theoretically and numerically the relative importance of the cooling - radiator mechanism, one has to take in account the evolution low of LPDE. The condition that rends the problem in one dimension is the thinness of the radiator (T depend only on x). The typical stages of a numerical simulation [4] are illustrated in figure (1).

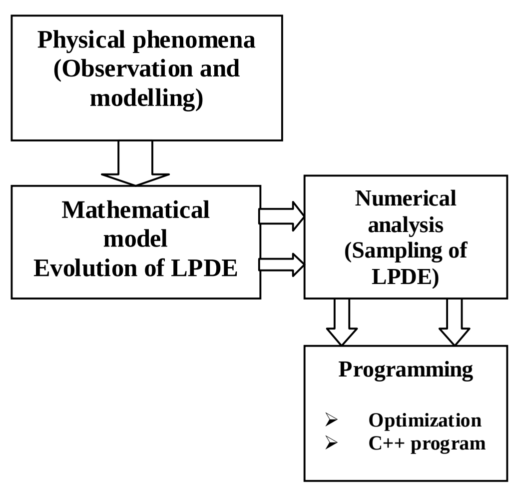

Fig (1): General model diagram

This paper is organized as follows. Section 2, we set in equations the proposed

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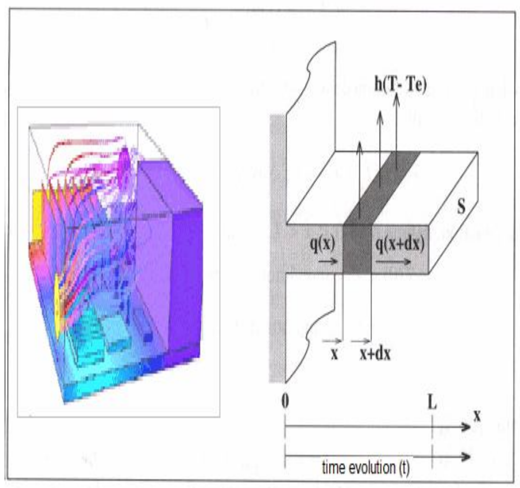
problem, while in section 3 all the steps which are necessary to analyze mathematically the equations are discussed. The results and all relevant discussions appear in section 4. Finally the main conclusions are summarized in section 5.

Non-Stationary Heat Equations

The geometry of the proposed system is showing in the figure (2).



(a) Radiator (b) One layer

Fig (2): Geometry of the model

The geometry is described by the length *L*, the transversal section *S*, the temperature generated by the processor T_0, and the ambient temperature T_a. To find the temperature distribution in the radiator T, it is supposed that the layer is sufficiently thin to consider our problem one-dimensional. The thermal equilibrium of an infinitesimal element dx is as follows[5]:

$$q(x)S - q(x+dx)S - h_c(x)(pdx)(T - T_0) = \rho C(\frac{\partial T}{\partial t}(Sdx)) \qquad (1)$$

The first term describes the internal flow of heat while the second term represents the external flow of heat. The last term of the above equation [6] indicates the loss heat by air, where q is the flux of heat by the unit of surface and time [Watt/m²], h_c is a surface coefficient of thermal transfer [W/(m² Kelvin)], p is the perimeter of the surface, ρ is the density [kg/m] and C is the mass of heat [Joule/(Kg Kelvin)].

Dividing equation (1) by $S\,dx$ for $dx \to 0$, because the radiator layer is sufficiently thin. The Fourier law expresses the proportionality between heat flux and gradient of temperature [7] as follows:

$$q(x) = -k\frac{\partial T}{\partial x} \qquad (2)$$

Where k is thermal conductivity [W/(mk)]. Finally, the distribution of temperature will be described by the ODE:

$$\frac{\partial T}{\partial t} - \alpha\frac{d^2T}{dx^2} + a(x)(T - T_a) = 0 \qquad \to (3)$$

Where $\alpha = \frac{k}{\rho C}[m^2/s]$ and $a(x) = \frac{h_c(x)pL^2}{kS}$

Equation (3) defined for $0 < x < L$, and then the boundary conditions must be added [8]:

$T(0) = T_0$ (Dirichlet type condition)

$\frac{dT}{dx}\Big|_{x=L} = 0$ (Neuman type condition)

The condition in x=L expresses the fact that the heat transfers through the last section q(L) is ignorable with respect to the initial section q(0).

Mathematical Analysis

The mathematical model developed in this paper consists of the following steps:

Step1: make eq (3) in dimensionless form. The dimensionless variables are defined as:

$$t^* = t(\frac{\alpha}{L^2}),\ x^* = \frac{x}{L} \text{ and } \theta^* = \frac{T - T_e}{T_e}$$

$$\theta^*(0,t^*) = (T_0 - T_e),\ \frac{\partial \theta^*}{\partial x^*}(1,t^*) = 0,\ \forall t^* \in [0, t^*_{Max}]$$

$$\frac{\partial \theta^*}{\partial t^*} - \alpha \frac{d^2\theta^*}{dx^{*2}} + a(x^*)\theta^* = 0 \qquad \rightarrow \quad (4)$$

$$\text{where} \quad 0 < x^* < 1,\quad 0 < t^* < t^*_{Max}$$

For notation simplicity the * can be removed and for reducing the calculation time $h_c(x)$ is considered as a constant.

Non-stationary equation (4) become stationary if $t^*_{Max} \rightarrow \infty$.

Step2: explicit discretization in space and time:

The first step in sampling consist of decomposition the segment $[0, L]$ in M intervals of length $h = \frac{L}{M}$. We obtain that the mesh consists of $M+1$ points of abscises $x_m = mh, \quad m = 0,1,...,M$. The idea of the numerical resolution is to find an approximation Θ_m of the exact solution $\theta_m = \theta(x_m)$. To obtain the discrete equation which take the values θ_m, the derivation operator must be sampled by using Taylor series expansion about x_m

$$[0,L] = \bigcup_{m=0}^{M-1}[x_m, x_{m+h}],\ x_m = mh, h = \frac{L}{M} \qquad (5)$$

m=1,2,…, M-1 that form a linear system with M-1 equations and M+1 unknowns.

By substituting the above conditions into eq (3), one can get the explicit discretization in time can be obtained by sampling the time interval [0..tmax].

$$[0, t_{Max}] = \bigcup_{n=0}^{N-1}[t_n, t_{n+\delta t}],\ t_n = n\delta t,\ \delta t = \frac{t_{Max}}{N} \qquad (6)$$

The values of solution $\theta(0,t)$ two points (x_m, t_n) will be $\theta^n_m = \theta(x_m, t_n)$. Using Taylor development [8] for two independence variables (x,t), we get,

$$\left.\begin{aligned} \theta_{m-1} &= \theta(x_m - h) = \theta(x_m) - h\theta'(x_m) + \frac{h^2}{2}\theta''(x_m) - \frac{h^3}{6}\theta'''(x_m) + O(h^4) \\ \theta_{m+1} &= \theta(x_m + h) = \theta(x_m) + h\theta'(x_m) + \frac{h^2}{2}\theta''(x_m) + \frac{h^3}{6}\theta'''(x_m) + O(h^4) \end{aligned}\right\}$$

One can obtain from [8]:

$$\frac{\partial\theta}{\partial t}(x_m, t_n) = \frac{\theta(x_m, t_{n+\delta t}) - \theta(x_m, t_n)}{\delta t} - \frac{\delta t}{2}\frac{\partial^2\theta}{\partial t^3}(x_m, \gamma) \quad \gamma \in [t_n, t_{n+1}]$$

(7)

and

$$\frac{\partial^2\theta}{\partial x^2}(x_m, t_n) = \frac{\theta^n_{m+1}(x_{m+h}, t_n) - 2\theta^n_m(x_m, t_n) + \theta^n_{m-1}(x_{m-h}, t_n)}{h^2} + O(h^2)$$

(8)

Substitute (7) and (8) into (4):

$$\frac{\theta^{n+1}_m - \theta^n_m}{\delta t} - \frac{\theta^n_{m+1} - 2\theta^n_m + \theta^n_{m-1}}{h^2} + a_m\theta = 0 \qquad (9)$$

where m=1,2, …, M-1.

As $\theta^0_m = \theta(x_m, t_0)$ is known, θ^{n+1}_m can be calculated by recurrence using the relation

$$\theta^{n+1}_m = (1 - a_m\delta t)\theta^n_m + \frac{\delta t}{h^2}(\theta^n_{m+1} - 2\theta^n_m + \theta^n_{m-1})$$

$$\theta^{n+1}_m = r\theta^n_{m+1}(1 - 2r - a_m\delta t)\theta^n_m + r\theta^n_{m-1} \qquad (10)$$

where $r = \frac{\delta t}{h^2}$. Under vector form:

$\theta^{n+1} = H\theta^{n}$ $\rightarrow$H linear operator from the equation (4).

$$\begin{pmatrix} 1-2r-a_1 & r & 0 & \dots & \dots & 0 \\ r & 1-2r-a_2 & r & \dots & \dots & 0 \\ 0 & r & 1-2r-a_3 & r & \dots & 0 \\ & & & & & \\ 0 & 0 & \dots & 0 & r & 1-2r-a_{M-1} \end{pmatrix} \begin{pmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \\ \\ \\ \Theta_{M-1} \end{pmatrix} = \begin{pmatrix} \Theta_0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}$$

Simulation Results

Consider the case that the following constants are taken into account for one layer of Aluminium, of section 4[mm] * 50[mm] and L=40[mm]. The temperature of the processor $T_0 = 46°C$ and the ambient temperature is $T_a = 20°C$. For Aluminium, the thermal conductivity is 164 $[W/(m\,k)]$ and the coefficient of heat transfer take the constant value $h_c = 200[W/(m^2\,k)]$. The variation of temperature distribution now can be studied on the Aluminium layer *range* $[0, L]$.

The numerical solution of the non stationary heat equation needs some modification compared with the stationary equation [1]:

1. the reference time $t_0 = L^2/\alpha$ is calculated with the value of thermal diffusion $\alpha = 68.2\,x10^{-6}[m^2/s]$;

2. in the initialization loop, it must be precise $\Theta(x, t=0)$; we considered two distinct conditions:

$$\Theta(x,0) = \Theta_0, 0 \le x \le 1 \; and$$
$$\Theta(0,0) = \Theta_0,\; \Theta(x,0) = 0, \quad 0 < x \le 1$$

3. the time step is given by the stability condition $\delta t \le \frac{h^2}{2}$.

The most important in the practical point of view is the convergence of the numerical solution Θ toward the exact solution θ when the discretization steps δt *and* h tend to zero, similar convergence studied in [9].

Figure (3) presents the simulation results of the time evolution in the radiator layer when the condition $\Theta(x,0) = \Theta_0, 0 \le x \le 1$ is applied. The non stationary solution is converged to the stationary solution when the grid points of discretization M=75.

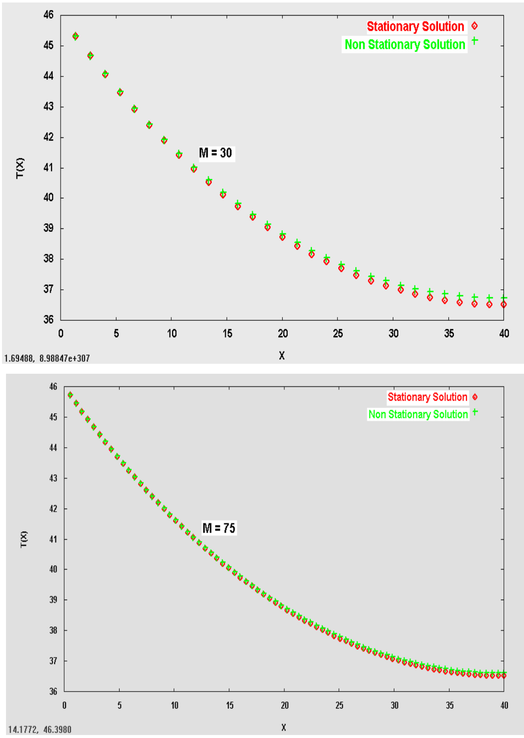


Fig (3): Comparison between stationary and non stationary heat transfer for different values of M (30, 75), hc=200, $\delta t = 0.5h$.

In both cases the diminution of the temperature is observed within the thin layer. On the other hand, the numerical calculations are valid because of the convergence toward the exact solution especially when the steps of discretization are taken 75.

Figure (4) shows the distribution of temperature in the radiator layer for the same values of given parameters as it is used for figure (3) except the value of $h_c = 20[W/m^2 s]$. Figure (4) also explains that the decreasing of the temperature is very low about $1°C$. In fact the value of h_c depends on the air flow and it is directly proportional with it.

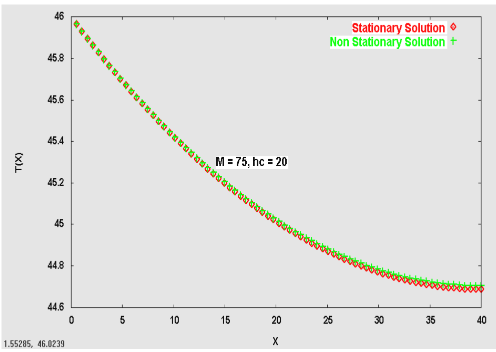


Fig (4): Comparison between stationary and non stationary heat transfer for different values of M (75), hc=20, $\delta t = 0.5h$ **.**

Conclusions:

In this paper, the simulation results show the efficiency of the proposed system in both temperature diminution and precision. In the case when the surface coefficient of the thermal transfer and the discretization steps are equal to 200, 75 respectively, our system grants the convergence (i.e., optimal solution).

Finally, the test results show that the effect of the other involved parameters (like; p, S, ρ) on the temperature distribution is very small.

The future work will be the study of non stationary heat equation in the computer radiator in 2D (T(x,y)). Another perspective of this research will be the measure of the produced error by approximation the continuous operators by discrete operators through the consistency concept.

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شیكاری ژمارەیی بۆ ساردكردنەوەی رادێتەری كۆمپیوتەر بە بەكارهێنانی هاوكێشەی گەرمی ناجێگیر

ئاری علی محمد ، نجم الدین واحد عبدالرحمان / كۆلێژی زانست-بەشی كۆمپیوتەر

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زانكۆی سلێمانی-هەریمی كوردستان/عیراق

پوختە

لەم تویژینەوەیەدا پەرەمان سەند بە نمونەیەكی بیركاری وە لێكۆلینەوەی هاوكێشەی گەرمی كە یەك گۆراوی سەربەخۆی تێدایە. ئە نجامی ئەم توێژینەوەیە بەراورد كرا بە مۆدێلی گوێزانەوەی گەرمی جێگیر.

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الخلاصة

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